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Implicit in (21) is an intrinsic speed for rock cutting,

$$c = \frac{k\tau_{o}}{\mu_{r}g} , \qquad (22)$$

which depends on all four properties of the rock but is independent of the jet. The failure criterion (21) can be rewritten in terms of the intrinsic speed as follows:

 $\tau = \tau_0 \left(1 + \frac{v}{c} \sin \theta\right) . \tag{23}$

When (23) is satisfied, the surface layer of grains is always on the verge of being shorn away.

It is now possible to show a posteriori that the thickness δ of the saturated front is indeed small. Since the two forms of (20) must be equal at $|n| = \delta$,

$$\delta = \frac{k(p_s - p_a)}{v \sin \theta} = \mu_r g \frac{p_s - p_a}{\tau - \tau_0},$$

where the second equality follows from (21). But (14) must be satisfied simultaneously, so

$$\delta = g \frac{\mu_r}{\mu_w} \frac{\tau}{\tau - \tau_o}$$

 δ is larger than g provided $\mu_r > \mu_w$, but not much larger unless τ is very nearly equal to τ_o , in which case permeability is unimportant anyway. Thus δ is small in a fine-grained rock.

Equations (23) and (14) give rise to a compatibility condition between the fluid and solid mechanics:

$$\mu_{\rm W}(p_{\rm s} - p_{\rm v}) = \tau_{\rm o}(1 + \frac{\rm v}{\rm c}\sin\,\theta) \quad . \tag{24}$$

It is interesting to inquire whether there are circumstances under which (24) cannot be satisfied. Consider the location $\sin \theta = 1$, where the right-hand side of (24) is maximum, and imagine v increasing indefinitely. Equation (8) suggests that p_s can also rise to any level if R becomes small enough. But equation (8) breaks down when $R \sim d$, and p_s cannot rise above the stagnation pressure P_o of the jet. If P_o lies below a critical value P_c given by

$$P_{c} = \frac{v_{o}}{\mu_{w}} (1 + v/c) , \qquad (25)$$